1. Which of the following is the first integral of the Euler-Lagrange equations if the integrand is of the form $F(y(x), y'(x))$?

a) $F = \text{constant}$
b) $y'F_{y'} = \text{constant}$
c) $F - y'F_{y'} = \text{constant}$
d) $F_{y'} - Fy' = 0$

2. Which of the following is a first integral that can be used to solve the Brachistochrone problem easily? (Note : C is an arbitrary constant)

a) $F - y'F_{y'} = C$
b) $F - y'F_{y'} = 0$
c) $F - y'F_{y'} + y''F_{y''} = C$
d) $F - y'F_{y'} + y''F_{y''} = 0$

3. The action integral for a vibrating string is $A = \int_{t_1}^{t_2}\int_{0}^{L} (\underbrace{\frac{1}{2}\rho\dot{w}^2}_{Term1} - \underbrace{\frac{1}{2}Tw'^2}_{Term2})dxdt$. Which one of the following statements is false?

a) Term 1 in the integrand is due to the kinetic energy of the string.
b) $\frac{1}{2}w'^2$ is the approximated extension in the string due to tension.
c) Sign of Term 2 in the integrand should be positive instead of negative in the action integral.
d) Term 2 is due to the strain energy in the string.

4. Which of the following should be minimized to find eigenvalue of a vibrating string?

a. Strain energy
b. Work potential
c. Rayleigh's quotient
d. Potential Energy

5. Which of the following is/are true in view of the minimum characterization of an eigenvalue problem?

A. In the vicinity of an eigenvalue, Rayleigh quotient is always less than or equal to it.
B. In the vicinity of an eigenvalue, Rayleigh quotient is always greater than or equal to it.

C. Minimum characterization of eigenvalue problem can determine eigenvalues but not mode shapes.

a) A,C
b) B,C
c) A
d) B

6. Noether's theorem in calculus of variations helps us to…

a) find the invariant co-ordinate transformations of an action integral.
b) simplify the minimization problem by easily solving Euler-Lagrange equation.
c) find the conserved quantities in a system.
d) All of the above

7. Rayleigh quotient for column buckling is ….

a) $\frac{\int_0^L EIw''^2 dx}{\int_0^L w'^2 dx}$
b) $\frac{\int_0^L EIw'^2 dx}{\int_0^L w'^2 dx}$
c) $\frac{\int_0^L EIw''^2 dx}{\int_0^L w''^2 dx}$
d) None of the above

8. Consider a particle of mass m moving in dimension under gravity g from time t_1 to t_2. Which of the following is the action integral/Hamilton of the system? Take position of the particle to be q ,velocity to be $\dot{q}$ and t to be time.

a) $\int_{t_1}^{t_2} \frac{1}{2} m\dot{q}^2 dt$
b) $\int_{t_1}^{t_2} mgqdt$
c) $\int_{t_1}^{t_2} (\frac{1}{2} m\dot{q}^2 - mgq)dt$

d) $\int_{t_1}^{t_2}(\frac{1}{2}m\dot{q}^2 + mgq)dt$

9. Check if the action integral obtained in the Question 8 is invariant under the two coordinate transformations :

1. $\overline{t} = t + \varepsilon_1$, $\overline{q} = q$

2. $\hat{t} = t$, $\hat{q} = q + \varepsilon_2$

a) Invariant under transformation 1 but not 2
b) Invariant under transformation 2 but not1
c) Invariant under transformations 1 and 2
d) Not invariant under transformations 1 and 2

10. If at least one of the coordinate transformations in Question 9 is invariant, use Noether's theorem to find out the symmetry/symmetries in the system and select the appropriate option from the following.

a. Only energy
b. Only linear momentum
c. Energy and linear momentum
d. None